Extending Capture Point to Dynamic Rigid Surfaces

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I. Background

Capture point is the point that a legged robot in the linear inverted pendulum mode (LIPM) should step in order to come to a complete stop given its current state. The concept of capture point has been widely used to plan footsteps for legged locomotion on stationary surfaces [1]–[3]. However, to the best of our knowledge, the extension of the capture point to legged locomotion on a dynamic rigid surface (DRS) (i.e., rigid surfaces that move in the inertial frame) [4] has not been investigated, which is the focus of this study.

II. Method

We considered a 3-D LIPM (with a point mass and a massless leg) moving on a DRS as a reduced-order dynamic model for a legged robot walking on a DRS (see Fig. 1). The capture point is derived based on the equation of motion of the LIPM with the DRS motion explicitly considered. To obtain an analytically tractable expression of the capture point, the following assumptions are considered: a) analogous to the stationary surface locomotion, the vertical height between the point mass, i.e., the center of mass (CoM) of the LIPM, and the support point is constant in the world frame and b) the support point velocity is constant in the horizontal directions of the world frame (e.g., during locomotion on an elevator or a ship in regular waters). Under these assumptions, the horizontal components of the dynamics of the LIPM moving on a DRS can be decoupled, which allows us to decouple the horizontal coordinates of the capture point.

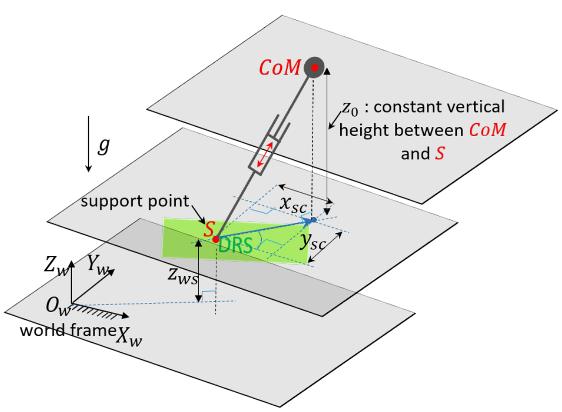

Fig. 1. Illustration of the 3-D LIPM moving on a DRS. The model is constrained to maintain a constant height z_0 between the CoM and the support point S on the DRS.

III. Preliminary Results and Discussion

The proposed derivation produces the following closed-form expression of the horizontal coordinates of the instantaneous capture point associated with a LIPM moving on a DRS:

$$x_{cap} := \sqrt{\frac{z_0}{(\ddot{z}_{ws} + g)}}\,\dot{x}_{sc} \text{ and } y_{cap} := \sqrt{\frac{z_0}{(\ddot{z}_{ws} + g)}}\,\dot{y}_{sc}, \quad (1)$$

where $\ddot{z}_{ws}$ is the acceleration of support point S in the world frame, x_{cap} and y_{cap} are the x- and y-coordinates of the capture point, respectively, and $\dot{x}_{sc}$ and $\dot{y}_{sc}$ are x- and y-coordinates of the CoM velocity relative to point S.

Based on the extended capture point, we have derived a footstep planner for quadrupedal walking on a DRS that explicitly considers the surface movement. We have also formulated an optimization-based controller based on a full-order robot model that provably tracks the desired motion while guaranteeing the feasibility of ground-contact constraints. The simulation results have demonstrated stable and efficient walking on the DRS under the proposed planner and controller.

IV. Conclusion

This study extends the capture point to a DRS that can be utilized to plan footsteps for legged locomotion on a DRS. By explicitly considering the DRS movement in the planning, it helps reduce the perturbations to the robot caused by the surface motion. Simulation results have shown the effectiveness of the proposed planning method for DRS locomotion. Our future work aims to validate the proposed method experimentally on a physical legged robot.

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References

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