Learned Linear Models for Online Motion Planning

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I. Introduction

Online motion planning remains a core challenge in controlling bipedal robots. To achieve computational tractability for online planning, state of the art methods use reduced order models such as LIP and SLIP [1], which limit the capabilities of complex robots to a low dimensional manifold. The single rigid body dynamics (SBRD) model is a more permissive model which allows arbitrary base motions up to actuation and friction limits, but whose nonlinearity poses problems for online planning. This model approximates the body of the robot as a single rigid body which obeys the Newton-Euler equations of motion. Here, we take inspiration from Koopman Operator theory to develop approximate *linear* equations of motion for the SRBD model, and formulate the motion planning problem in a convex optimization framework to enable online motion planning.

II. Koopman Operators

Koopman Operator theory as applied to control seeks to approximate a nonlinear dynamical system $\dot{x} = f(x, u)$ as a linear system of the form:

$$\dot{z} = Az + Bu \tag{1}$$

where z is a vector of scalar-valued, continuous functions of x called observable functions [2]. Including the elements of x as elements of z allows for the natural expression of costs and constraints in this new Koopman space. Planning can then be formulated as a quadratic program, a popular form of optimization for online control due to the reliability and speed of modern solvers.

III. Approach

Linear models are identified by sampling the state space of the SRBD model and calculating the dynamics of z at each point analytically, then applying a least-squares fit to this dataset to find A and B which best explain the single rigid body dynamics in Koopman space. Trajectory optimization is performed online in a model predictive control (MPC) fashion by solving a quadratic program with the integration of (1) as a dynamics constraint.

Preliminary simulation experiments controlling an underactuated planar rigid body with a trivial Koopman space of $z = \begin{bmatrix} x & 1 \end{bmatrix}^T$ show the Koopman MPC controller is able to generate stable walking motions. In fact, MPC using this model is more robust to disturbances and permits larger steps than MPC using a linearization of the Newton-Euler equations.

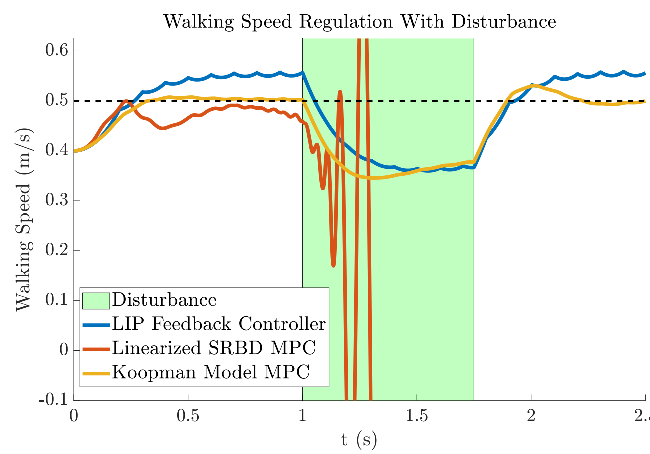

Fig. 1. The Koopman MPC model is best able to reject a disturbance force of 13% of the robot's weight to maintain the desired walking speed

Our hypothesis for why a simple Koopman model outperforms the linearization is that the Koopman model reflects the true dynamics more accurately away from the linearization point. Additionally, choosing $z = \begin{bmatrix} x & 1 \end{bmatrix}^T$ rather than a choice of z with additional functions of x may have performed better due to issues of closure under the Koopman operator. A closed, finite dimensional approximation for the Koopman operator does not exist for most systems [3], including the SRBD model. Adding additional terms to the Koopman representation may therefore corrupt A and B with spurious correlations.

We are currently implementing the Koopman MPC controller on a five link planar walker in simulation to test the applicability of this framework to robots with massive legs and rigid body impacts. Additional work is also needed to find an optimal Koopman representation of the 3D SRBD model and resolve the closure challenges which limit the performance of higher dimensional Koopman models.

References

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