Sliding Cone Control for Hopping on Rough Terrain

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I. Introduction

Agile locomotion on rough terrain is a very challenging topic. To traverse through complicated ground surface, a robot has to be agile enough in order to adapt to ground uncertainties and response to unpredictable disturbances, while accurately executing the locomotion tasks. In order to hop without stumbling, the robot needs to modify its energy such that it can reach different desired apex heights and forward velocities. In our work, a novel Sliding Cone Control (SCC) was adopted to regulate the leg compression in a robust limit cycle following manner, facilitating integration of high level controllers for hopping tasks even on a non rigid ground surface.

II. Sliding Cone Control (SCC)

Inspired by the geometry, we design the SCC by first considering the desired Control Lyapunov Function to be a curved surface that looks like a Mexican hat (Fig. 1(c)), with the trough as a close curve represented by the desired limit cycle of a second-order system. We expect the state $[\boldsymbol{x}, \frac{1}{2}s^2]^T$ to slide down to and stay on the limit cycle. We also choose $V = \frac{1}{2}s^2$ as the Lyapunov function candidate. From this choice of Lyapunov function, we need to choose a sliding surface to be the intersection of an inverted cone $s(x)$ and $s = 0$ (Fig. 1(b)). Consider the estimated system dynamics,

$$\dot{\boldsymbol{x}} = \omega \boldsymbol{J}\boldsymbol{x} + (-2\hat{\zeta}\hat{\omega}x_2 + f/\hat{\omega})\boldsymbol{e}_2, \tag{1}$$

where $\hat{\zeta}$ and $\hat{\omega}$ denote the estimated parameters with estimation error fulfilling the following constraint:

$$\left| -2\zeta\omega + 2\hat{\zeta}\frac{\hat{\omega}^2}{\omega} \right| \leq F, \tag{2}$$

where F is some known function. A control input

$$f = 2\hat{\zeta}\hat{\omega}^2 x_2 - k_{SCC}\hat{\omega}sgn(x_2)sgn(s) \tag{3}$$

with

$$k_{SCC} \geq \frac{\omega}{\hat{\omega}}\left(F|x_2| + \eta\frac{r\|\boldsymbol{x}\|}{s_0|x_2|} \right), \tag{4}$$

where $s_0 > 0$, and $\eta > 0$, will stabilize the system to $\|\boldsymbol{x}\| = r$. And by choosing the sliding surface s as

$$s = -s_0\left(1 - \frac{\sqrt{\boldsymbol{x}^T\boldsymbol{x}}}{r}\right), \tag{5}$$

and defining the Control Lyapunov Function as $V = \frac{1}{2}s^2$, we can proof that the system will converge to the state $\|\boldsymbol{x}\| = r$. More specifically, the system tracks an asymptotically stable periodic orbit according to the definition [1].

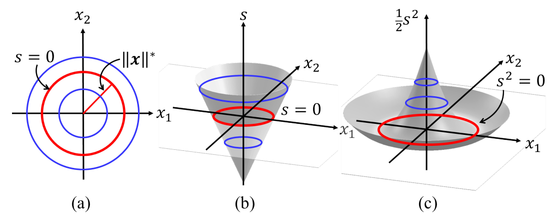

Fig. 1. Schematic diagrams of SCC sliding surface

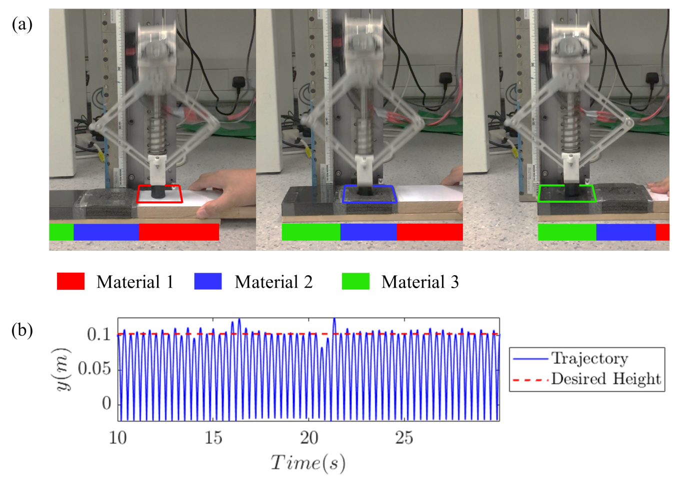

Fig. 2. (a) Snapshots of test carried out on a 'ground' composed of three types of materials with different stiffness. The 'ground' was pulled to the right hand side slowly, allowing the system to hop on three different grounds. (b) Height profile

III. Results and Discussion

Fig. 2(a) shows the snapshots of the test carried out on a 'ground' composed of different type of materials with varying stiffness. During the test, the 'ground' was gradually pulled to the right while the robot was hopping. Material 1 was a hard wooden block, material 2 was a soft packing foam block, and material 3 was a 3D-printed TPU95 elastic block. Fig. 2(b) showed the experimental results of the test. Without adjusting the parameters in the SCC controller, the hopping height was self-maintained.

From the result, we have seen that the controller offers the ability of height control even if there is change in the ground stiffness. In the coming future, we will try to implement the controller on a planar hopping robot to see how the controller facilitate high level control of locomotion tasks.

References

[1] H. K. Khalil, "Nonlinear systems". *Upper Saddle River, NJ: Prentice Hall*, 2002.

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