# Constant speed gaits should work across all speeds

Nick Gravish<sup>1</sup>, Dan Zhao<sup>2</sup>, Glenna Clifton<sup>3</sup>, Brian Bittner<sup>2</sup>, Shai Revzen<sup>2</sup>

1: UC San Diego, 2: U. Michigan, 3: U. Portland



6 leg body length



#### **Model**

• Leg contact position and velocity in body frame is  $q_k(t), \dot{q}_k(t)$ 

### a. Viscous friction

• Viscous contact forces yield velocity dependent forces,  $F_k \propto \dot{p}_k$  , and thus a change of



#### Introduction

Many multi-legged robots and animals move in friction dominated regimes where contact forces dissipate momentum rapidly. For example the multi-pod robot shown above slips during approximately 50% of its gait, dissipating energy.

Predicting how locomotion kinematics vary as the gait pace of legged walking changes can be useful for designing new gaits, however modeling these frictional regimes can be challenging.

In this work we propose that constant speed gaits yield a simple scaling relation dependent upon the pacing change to the gait,  $\alpha$ .

We demonstrate this simple scaling in the case of viscous, static Coulomb, and dynamic Coulomb friction contacts.

Surprisingly, regardless of the type of friction generated by contacts, any constant speed walking gait will work at all average speeds

• Body and world frame are related by the transform,  $g_{i}$ , and thus contact position is  $p_k = gq_k$  and velocity  $\dot{p}_k = \dot{g}q_k + g\dot{q}_k$ 

• A pacing change of gait,  $q'_k(t) = q_k(\tau(t))$ yields the new contact velocity  $\dot{q}'_k = \alpha \dot{q}_k$ where  $\alpha(t) := \frac{d}{dt}\tau(t)$ 

• Gaits produce constant speed body motion and thus contacts are in force balance

### Conclusion

 Speed changes to quasi-static gaits yield new quasi-static gaits across friction regimes

• Any constant speed gait discovered while moving slowly can be used at any speed. Thus, constant speed gaits work the same at all speeds. pacing produces,  $F'_k = \alpha F_k$ 

• Since the original gait was in force balance the new gait remains in force balance and thus the body movement is scaled by  $\alpha$ • A change of gait pacing by  $\alpha$  will proportionally increase speed by  $\alpha$  in this friction regime

# **b. Static Coulomb friction**

• With all feet in static contact  $\dot{p}_k = 0$  and thus  $\dot{g}q_k + g\dot{q}_k = 0$ 

• Under a change of gait pacing  $\alpha$  the scaling of body speed by  $\dot{g}'_k = \alpha \dot{g}$  preserves the contact condition and thus body speed scales by  $\alpha$ • A change of gait pacing by  $\alpha$  will proportionally increase speed by  $\alpha$  in this friction regime

## c. Dynamic Coulomb friction

• With all feet sliding under dynamic Coulomb friction  $F_k = -\mu_k N_k \dot{p}_k \|\dot{p}_k\|^{-1}$ 

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• Our results suggest that there's a significant ad-

vantage to learning constant speed gaits as they

can then be applied over a large range of speeds

by following the same set of body shapes using

proprioceptive feedback

• Consider a speed change that preserves the time-dependent normal force  $N'_k(t) := N_k(\tau(t))$ 

• Scaling body velocity by  $\alpha$  would produce foot velocities  $\dot{p}'_k = \alpha \dot{p}_k$  and would thus pro-

duce the same foot contact forces  $F'_k = F_k$ 

• A change of gait pacing by  $\alpha$  will proportionally increase speed by  $\alpha$  in this friction regime