Foot Tipping Allowance in Legged Balancing with Conditional Constraints in Optimization

William Z. Peng
Department of Mechanical and Aerospace Engineering
New York University
Brooklyn, NY, USA

Hyunjong Song
Department of Mechanical and Aerospace Engineering
New York University
Brooklyn, NY, USA

Joo H. Kim
Department of Mechanical and Aerospace Engineering
New York University
Brooklyn, NY, USA
Email: joo.h.kim@nyu.edu

I. Background

Tipping occurs frequently in legged systems, including during stable gait [1]. However, tipping is often not considered explicitly within existing motion planning frameworks because there is no conventional trajectory optimization method for simultaneous contact event detection and planning. Here, an optimization framework with conditional constraints is established and validated for direct collocation in trajectory optimization for tipping allowance in legged balancing.

II. Optimization with Conditional Constraints

Direct methods for trajectory optimization based on nonlinear programming, such as direct collocation methods, require *a priori* knowledge about contact changes, such as their timing, to produce feasible motions consistent with both system dynamics and evolving contact interactions [2]. When direct methods are applied to evolving contacts, they result in unrealistic anticipatory behaviors. In the proposed framework, contact constraints are selectively disabled or enabled whenever certain kinematic and kinetic conditions are triggered, by relaxing their bounds to extreme values or re-imposing the original bounds, respectively. This method avoids discontinuities in the gradients and complementarity constraints.

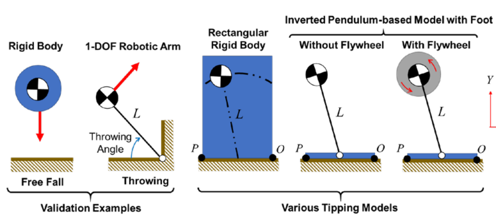

Fig. 1. Models for free fall, throwing, and tipping. All tipping models share the same base of support [–0.2 m, 0.2 m] and tipping edge *O*. The length *l*, flywheel moment of inertia, and mass of each model are 0.75 m, 0.1 kg-m^2, and 10 kg, respectively. The mass of each rigid body is concentrated at its COM.

This framework was validated by the simulation of free-fall and throwing trajectories with unknown collision timing, which is not possible with conventional trajectory optimization. For free fall, the conditional constraint and trigger condition are a basic above-ground kinematic constraint and condition. For throwing, a joint angle kinematic constraint and condition are used to consider internal collision of a robotic arm with its mechanical joint stops. Without conditional constraints, maximization of the end effector velocity of a 1-DOF robotic arm at a throwing angle of $\pi/4$ within a range of motion of $[0, \pi/2]$ (Fig.1) yields 1.48 m/s, with the end effector coming to a stop at $\pi/2$. With conditional constraints, the end effector velocity is 2.00 m/s and 1.92 m/s at $\pi/4$ and $\pi/2$, respectively, without any anticipatory stopping behavior.

For the case of tipping and then foot collision (i.e., return to full contact), tipping is triggered when the center of pressure reaches the tipping edge *O* and a ground reaction moment threshold is met, upon which a kinematic constraint at *P* is disabled such that the system tips about *O*. Collision is detected when foot angle, foot angular velocity, center-of-mass (COM) position, and COM velocity thresholds are met such that a collision will occur without further actuation. The pre-collision system state may be used as the initial conditions of another trajectory optimization for a post-collision trajectory.

III. Results and Discussion

Balanced regions are computed with an optimization-based method [3] to demonstrate the proposed framework by quantifying the effect of tipping. Over a common base of support, the balanced region areas of the rigid body, inverted pendulum-based model (IPBM) without a flywheel, and IPBM with a flywheel with tipping allowance are 98.8%, 108.1%, and 121.9% that of the IPBM without a flywheel in full contact, respectively (Fig. 2). The balanced region of the IPBM without a flywheel with tipping is verified in simulation over the COM *X*-position range [–0.15 m, 0.15 m]. The proposed framework may be extended to rolling and slipping in legged systems.

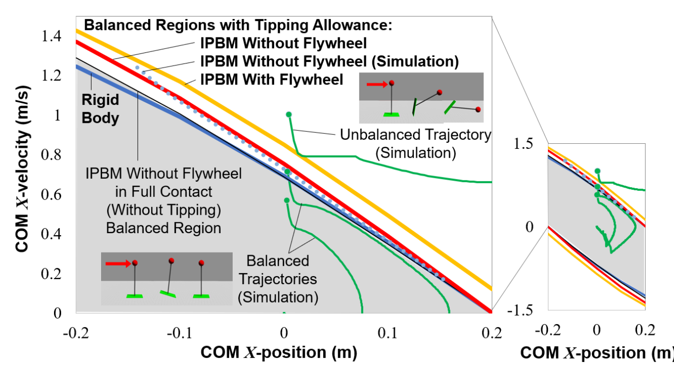

Fig. 2. Balanced regions with tipping allowance and for full contact (shaded)

References

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This work was supported in part by the U.S. National Science Foundation (CMMI-1436636 and IIS-1427193).