Bipedal Walking on Constrained Footholds: Momentum Regulation via Vertical COM Control

Min Dai, Xiaobin Xiong and Aaron Ames

I. Summary

One of the central goals of the bipedal walking robot community is to develop humanoid robots that can locomote diverse terrain types, including flat surfaces, ascending and descending stairs, and discrete stepping stones with height variations, which requires locomoting in environments with *constrained footholds*. This has been studied via the hybrid zero dynamics framework with gait library and control barrier function [1], [2]. However, generating gait library is a nontrivial task and requires substantial computing power. In our work [3], we presents a framework that enables dynamic walking on constrained footholds with reactive online planning.

II. Method

The key observation underlying this approach is to consider the hybrid dynamics of the underactuated COM dynamics consisting of both continuous dynamics (encoding the COM forward dynamics and angular momentum) and discrete dynamics (representing the impact equations). With this representation, we regulate the post-impact angular momentum via the *vertical COM velocity* leveraging the discrete (impact) dynamics of walking:

$$\mathcal{HZ} = \begin{cases} \begin{cases} \dot{p}^x_{\text{com}} = \frac{1}{p^z_{\text{com}}}(L_y + p^x_{\text{com}}\dot{p}^z_{\text{com}} - L_{\text{com}}) \\ \dot{L}_y = g p^x_{\text{com}} \end{cases} & \mathbf{x} \in \mathcal{D} \setminus \mathcal{S} \\ \begin{cases} p^{x+}_{\text{com}} = p^{x-}_{\text{com}} - p^{x-}_{\text{sw}} \\ L^+_y = L^-_y + p^{x-}_{\text{sw}}\dot{p}^{z-}_{\text{com}} - p^{z-}_{\text{sw}}\dot{p}^{x-}_{\text{com}} \end{cases} & \mathbf{x}^- \in \mathcal{S} \end{cases}$$

where $p^{x,z}_{\text{com}}$ is the COM x, z position relative to stance foot, L_{com} is the y-component of robot's mass-normalized centroidal momentum, L_y is the mass-normalized angular momentum about the stance pivot, and super scripts $-,+$ represent pre and post impact states.

To apply this methodology to the full-order robot dynamics, we use the underactuated LIP model to approximate the continuous dynamics of p^x_{com} and L_y of the robot during stance, which determines the step duration and desired post-impact momentum based on *orbital energy* in the beginning of the next step. The desired vertical COM velocity is then realized via the online optimization of vertical COM trajectory under the kinematic constraints, which then creates vertical COM oscillation. The desired trajectories of the torso and swing foot are constructed with polynomials. Finally, a quadratic program based controller is applied for trajectory tracking [3, Algorithm 1]. We realize the proposed approach on two robots, AMBER and Cassie, to demonstrate generality. Under the momentum regulation, both robots can walk stably in various scenarios with different types of foothold constraints, as shown in the accompanying video [4].

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The authors are with the California Institute of Technology. {mdai, xxiong, ames}@caltech.edu

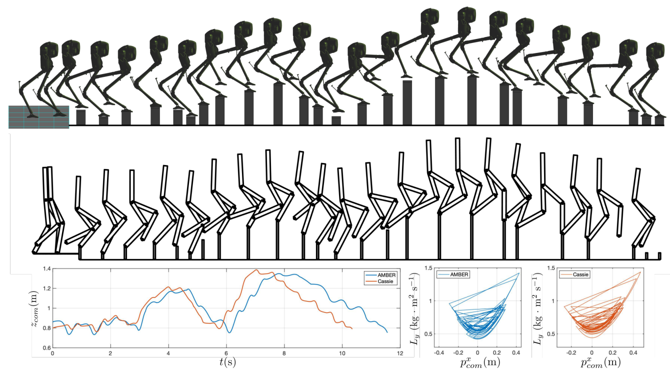

Fig. 1. The gait tiles for the walking of Cassie and AMBER on random-located stones, the vertical trajectory of the COM in global frame w.r.t. time, and the trajectories of the underactuated states in the phase plots.

III. Results

A variety of periodic desired footholds with distance (0.1m to 0.8m) and height (−0.2m to +0.25m) are first tested on both robots, resulting in the underactuated dynamics converged to periodic orbits with COM forward speed ranging from 0.4m/s to 1.6m/s (with different orbital energy level). For the stepping stone problem with randomly varying stone distance between 0.2m and 0.7m and height between ±0.25m. Both robots can successfully walk over the same settings of the stones. Fig. 1 demonstrates one comparison in the simulated walking. Despite the model difference among the two robots, the behavior of underactuated dynamics in the state space is very comparable: the resulting trajectory converged to a similar set as both simulated walking is designed based on same orbital energy level.

References

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[2] Q. Nguyen, A. Agrawal, X. Da, W. C. Martin, H. Geyer, J. W. Grizzle, and K. Sreenath, "Dynamic walking on randomly-varying discrete terrain with one-step preview." in *Robotics: Science and Systems*, vol. 2, no. 3, 2017.

[3] M. Dai, X. Xiong, and A. Ames, "Bipedal walking on constrained footholds: Momentum regulation via vertical com control," 2021.

[4] Supplementary video, https://youtu.be/BKx2F9STyVA.