Tractability of Stability-Constrained Trajectory Optimization

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I. Introduction

The effectiveness of open-loop stable walking has been demonstrated by several authors, however, given the absence of any feedback correction, the control design for open-loop stable walking is nontrivial. Previous attempts have cast the problem of open-loop stability in the framework of optimization, however, these approaches often encode stability in terms of some physical characteristic of the system, in particular, in terms of some measure of the system's energy. However, it is often challenging to design a robust indirect measure for stability. Recently, work has been undertaken to allow for direct constraints on stability that constrain the spectral radius of the linearized return map associated with a periodic solution to the system. However, finding the eigenvalues of the return map in the optimization framework is a challenge in its own right. Previous work on this problem has focused primarily on affine and symmetric matrix functions, which lead to a convex optimization program, however, the return map in general does not fit these criteria. Mombaur et. al. [1] address eigenvalue optimization using a Nelder-Mead type direct search method, however, direct search methods do not inform the search with the gradients of the solution landscape, and thus such methods can be prohibitively slow for high degree-of-freedom systems.

II. Methods

We solve the eigenvalue optimization problem with direct collocation, which is a fast and scalable nonlinear optimization technique with support for exact derivatives. To this end, we develop and implement various formulations of the eigenvalue problem and evaluate them at stabilizing the chaotic dynamics of the double pendulum. For evaluating the stability of limit cycles, we implement and compare two different stability evaluation techniques in direct collocation, one based on the spectral radius of the Poincaré map associated with the limit cycle, and the other based on the spectral radius of the system's monodromy matrix.

III. Results and conclusions

We have found stability evaluation based on the system's monodromy matrix to be better than the Poincaré map based method. This is because of numerical inaccuracies, as the calculation of the Poincaré map is usually numerical.

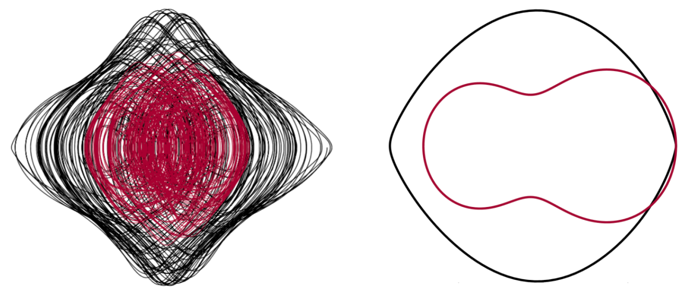

Fig. 1. (a) The chaotic dynamics of a double pendulum. (b) Stabilized double pendulum dynamics obtained with stability-constrained optimal control.

Our first approach to constraining the spectral radius of the mondoromy matrix is based on constraining its Frobenius norm. Though this method is reliable and fast, the Frobenius norm is a rather conservative bound on the spectral radius. To formulate a tight bound on the spectral radius we need to compute the dominant eigenvalue inside optimization. We formulate the eigenvalue optimization problem as a constraint satisfaction problem first based on power iteration, which is a successful numerical algorithm for computing eigenvalues, and then based on the definition of the eigenvalue: $Av = \lambda v$. The former works based on the assumption that the ratio $|\lambda_1/\lambda_2|$ is strictly greater than 1, which renders it useless for real matrices where the dominant eigenvalue is complex. The eigenvalue definition based method does not suffer from this limitation, but the constraint landscape defined by $Av = \lambda v$ is discontinuous, and hence difficult to search over.

We also formulated the eigenvalue problem in terms of eigendecomposition and Schur decomposition, and found these formulations to be more general in terms of their applicability, albeit slower. In the future, we we hope to develop better formulations of the eigenvalue problem in direct collocation, as well as extend our methods to work for hybrid limit cycles.

References

[1] K. D. Mombaur, H. G. Bock, J. P. Schloder and R. W. Longman, "Human-like actuated walking that is asymptotically stable without feedback," Proceedings 2001 ICRA. IEEE International Conference on Robotics and Automation (Cat. No.01CH37164), 2001, pp. 4128-4133 vol.4, doi: 10.1109/ROBOT.2001.933263.