A simple model of energy cost for running on slopes

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I. Introduction

The metabolic cost of running increases for steep upward or downward slopes, with substantial cost on level ground [1]. Steep slopes are largely explained by work performed against gravity, with different proportionalities (via work efficiencies) for positive work going uphill and negative work downhill. That leaves the cost for level ground unexplained, since there is net zero work against gravity. This also occurs for the classic spring-mass (or spring-loaded inverted pendulum, SLIP) model of running, which explains how series elasticity eliminates the need for active work on flat ground [2]. But it does not explain why humans expend metabolic energy at all, and why cost is minimized at shallow downhill slopes. Here, we augment the spring-mass model with minimal features that may explain the metabolic cost of sloped running. We propose energy is expended partly to restore dissipative losses, and also for a cost of rapid force production [3]. We aimed to reproduce running cost on sloped ground to understand energetic cost on the level.

II. Methods

We used dynamic optimization to determine how energy cost changes with ground slope in a simple model of running. We augmented the spring-mass model with two features important for the energetic cost of running: active actuation and passive dissipation (Fig. 1A). The actuator produced axial force and displacement in series with the spring, and could actively perform negative and/or positive work during stance. We modelled passive dissipation with losses from foot-ground collision at contact, and tendon hysteresis via parallel damping (Fig. 1A). Dissipation was important for overall work balance per periodic stride: the net work performed by actuation plus the net work gained from gravity on a slope equals the total negative work of passive dissipation. Actuation was optimized to minimize energy cost from active actuator work (scaled with proportionality constants for positive and negative work) plus a *force-rate* cost (the time-derivative of force production), based on empirical observations of energy cost for rapid force production in muscle [3]. We varied individual parameter values to assess effects on the model's cost vs. ground slope and match with human running metabolic data.

III. Results & Discussion

Similar to empirical observations [1], our model reproduces total energy cost increasing toward asymptotes proportional to the net work of steep ground slopes (see work efficiency asymptotes, Fig. 1B). However, cost gradually diverges from asymptotes at shallow slopes, and is substantially greater than zero, even on level ground. Steep slopes have straightforward cost explained by work (Fig. 1C), with positive work dominating for upward slopes, and negative for downward slopes [1]. For shallow slopes, there are two additional contributions to energy cost. First, there is small but meaningful passive dissipation requiring restoration by active work at most slopes. At -8% (descending), dissipation nearly matches the work of gravity; thus, there is minimal active work and minimum cost, as with humans. Second, the force-rate cost also increases expenditure. There is direct cost from force-rate itself, where rapid force production costs energy, but also indirect cost where active positive and negative work are performed (even at zero slope) to avoid even higher force-rate costs. Force-rate cost is particularly high at shallow slopes, because the optimization uses relatively impulsive force profiles (of high force and brief duration) to increase passive dissipation that nearly matches work done by gravity. There are surely other factors to metabolic cost, but our model suggests that positive work to overcome dissipation and gravity, and a cost due to force-rate, may both determine much of the cost of running at any slope, including level ground.

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References

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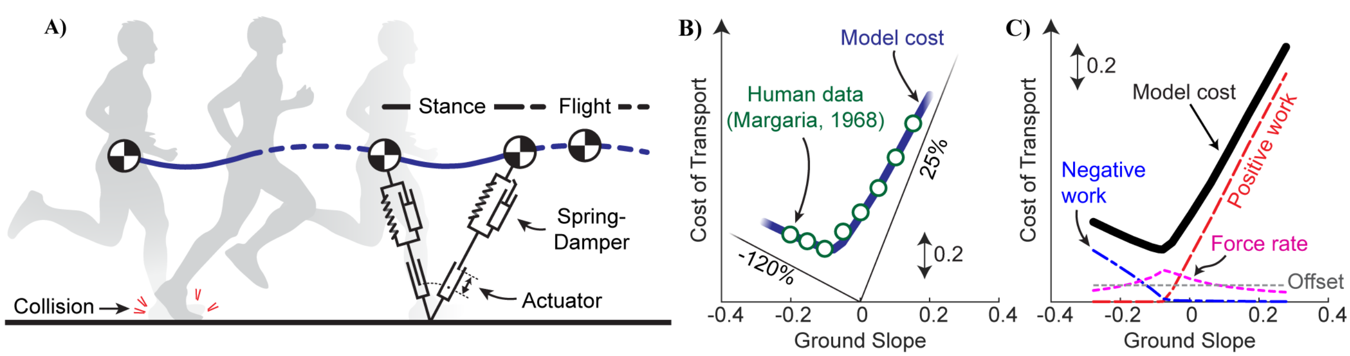

Fig. 1. Modeling energy cost of running. A) Spring-mass model with point-mass body and spring, augmented with damper and collision for passive dissipation, and a series actuator. B) Cost of running on slopes for humans [1] and model. C) Constituents of model energy cost: active positive/negative work, force rate, and a constant offset (similar to a human's resting energy expenditure).