Optimal Footstep Locations and Forces During Locomotion with Bilinear Alternation

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***Abstract*—Many trajectory optimization formulations for legged robots are inherently nonlinear due to the dynamic constraints that include bilinear terms from the cross product of the footstep locations and ground reactions forces. In this work, we suggest the use of bilinear alternations to simultaneously solve for the optimal footstep locations and forces for a single rigid body model based on the MIT mini cheetah.**

Keywords*—*Bilinear alternation, trajectory optimization, footstep planning

I. Introduction

Selecting footsteps locations and ground reaction forces (GRFs) for robust locomotion is crucial for legged robots. While the single rigid body model (SRBM) significantly simplifies the robot dynamics, trajectory optimizations are still nonconvex and nonlinear due to the bilinear products of contact locations and GRFs. To circumvent this issue, the footstep locations can be prespecified before optimizing over the GRFs [1] or regularized within a nonlinear optimization [2]. However, the success of these methods rely heavily on heuristics that must be determined by the user. In this work, we use bilinear alternations to simultaneously solve for the footstep locations and GRFs by iteratively solving two quadratic programs (QP). This method does not rely on any heuristics or initial guesses to produce the same solution.

II. Methods

We formulate two nearly identical trajectory optimizations as two QPs, where one QP solves for the GRF given the footstep locations (as in [1]) and the other solves for the footstep locations given the solutions for the GRFs until convergence. Both QPs also solve for the position, orientation, velocity and angular velocity of a simplified SRBM based on the MIT Mini Cheetah over a fixed trot gait. The same quadratic cost function, bounds, and linear constraints are used. We use a direct transcription method to formulate the QPs in Casadi [3], which are solved using Gurobi [4].

III. Results and Discussion

With few iterations, the cost of the optimization decreases dramatically, and the center of mass of the robot tracks the desired velocities far more closely (Fig. 1). The iterative process was initialized with the Raibert heuristic, as in [1], but deviated significantly from it after several iterations. The alternating formulation was not exploited for efficiency, and each QP iteration was solved in roughly 10 ms in MATLAB on a typical laboratory desktop computer.

Future work will compare the solve times and reliability between the bilinear alternation method and a non-linear trajectory optimization which showed unequaled robustness in practice on the MIT Mini Cheetah quadruped.

References

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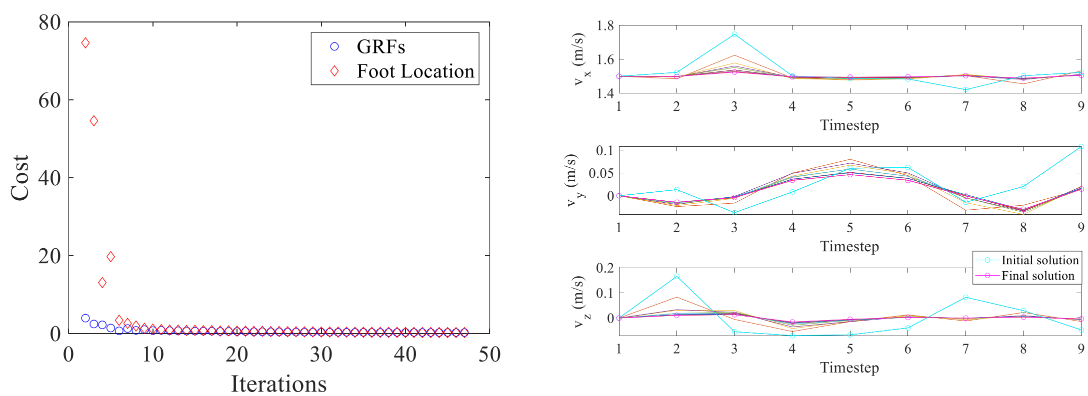

Fig. 1. Optimization cost (as detailed in [1]) over iterations (left) and comparison of initial and final center of mass velocities from iterated solutions (right).