A Hybrid Differential Dynamic Programming Method for Constrained Trajectory Optimization

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I. Introduction

Trajectory optimization (TO) has been a widely-used framework in robotics and control communities, especially in robot locomotion. For example, when planning a collision-free and dynamical feasible trajectory for a quadrotor or generating a control sequence for a bipedal walking robot while respecting the input limitations, the numerical optimization-based approach could be a powerful tool. To solve these involved nonlinear programming (NLP), some off-the-shelf NLP solvers like SNOPT are often employed due to their inherited features of robustness, easy-to-deal-constraints and easy-implementation. However, they are too slow sometimes and require high computation power. By contrast, Differential Dynamic Programming (DDP) gives a faster alternative way to solve the optimization problems by exploiting the Markov structure of the system dynamics.

Unfortunately, unconstrained DDP methods can hardly be applied to legged robots directly. Actuator saturation, joint limits, limitations of torso orientation, and friction cone need to be treated as inequality constraints in the NLP problem. It also cannot handle the infeasible initial start. Although some researchers have applied the DDP method and its variants to solve the trajectory optimization problem, they either incorporated DDP with state/input inequality constraints through penalty/active-set methods or extended DDP-like algorithms to a multiple shooting framework. Only few of them integrated these modifications together to systems with with complex nonlinear constraints. Therefore, to utilize the advantage of enabling an infeasible initial start from the multiple shooting framework and involve nonlinear state/input constraints simultaneously, we propose a hybrid Differential Dynamic Programming method for the constrained TO problems.

II. Methods

Our approach has two stages. The first stage rapidly solves the constrained trajectory optimization problem to get a low-precision solution using Augmented Lagrange method. In the second stage, we further post-process the resulting solution using an interior point method.

Following the work of [1], we firstly turned the constrained optimization problem into unconstrained one by wrapping DDP up with Augmented Lagrange method, named AL-MSDDP. After each iteration, including a pair of backward pass and forward rollout, the dual variables are then updated. Augmented Lagrange method is suitable for use with the multiple shooting settings without being unstable during iterations, which is important for DDP. Although the coarse solution from AL-MSDDP might not satisfy the tolerance of constraints or might be infeasible due to numerical ill-conditioning and slow tail convergence of penalty methods, it is still a good initial guess to warm start the second stage.

To refine the coarse solution obtained in the first stage, we applied the relaxed log barrier function method to DDP [2], named RLB-MSDDP, to approximate the optimization problem [3]. RLB-MSDDP shares the same shooting phases and constraints with AL-DDP. Initialized by the solution from AL-DPP, RLB-MSDDP can obtain a refined solution with an improved tolerance of constraints satisfaction and can even adapt an infeasible trajectory to be feasible. Such an adaptation is our main contribution, allowing us to get feasible state/input trajectories and reserve the linear feedback policy in the form of $\mathbf{u} = \mathbf{u}_{ref} + \mathbf{K}(t)(\mathbf{x} - \mathbf{x}_{ref})$ to improve robustness in the dynamic locomotion of legged robot.

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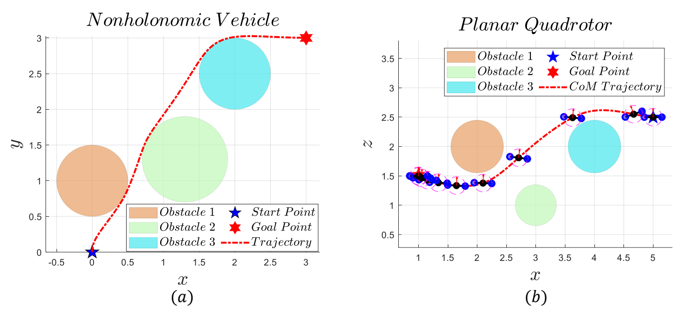

Fig. 1: Illustrating examples on generating collision-free trajectories. (a) Nonholonomic Vehicle; (b) Planar Quadrotor

III. Initial Simulation Results

We verified our proposed approach on two underactuated systems, a nonholonomic vehicle and a planar quadrotor. The vehicle was expected to move from $\mathbf{x}_0 = [0; 0; 0]$ to $\mathbf{x}_f = [3; 3; \pi/2]$. The optimal trajectory minimized the control effort and deviation from the goal state. AL-MSDDP took 3 iterations and then RLB-MSDDP took 49 iterations to find a collision-free optimal trajectory without violating input limitations as shown in Fig. 1 (a). For the planar quadrotor, AL-MSDDP stopped at the 2nd iteration and returned an infeasible trajectory. RLB-MSDDP took another 29 iterations to converge to an optimal solution while eliminating infeasibility as shown in Fig. 1 (b).

IV. Discussion

We have tested our algorithm in some classical underactuated systems. It is natural to extend our method to legged robot systems. For example, with predefined contact sequences and step timings, our approach is supposed to generate different gaits including walking, trotting and hopping [4] with a time-varying linear feedback policy, which is our future work.

References

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