# Momentum-Aware Planning Synthesis for Dynamic Legged Locomotion

Ziyi Zhou, Bruce Wingo, Nathan Boyd, Seth Hutchinson, Ye Zhao Georgia Institute of Technology, Atlanta, GA, USA {zhouziyi, bwingo, nboyd31}@gatech.edu

## I. INTRODUCTION

Dynamic motion generation for legged robots remains challenging due to the non-cyclic contact and highly nonlinear rigid body dynamics. Trajectory optimization (TO) with embedded contact dynamics generates elegant motions, yet solving the associated nonlinear program (NLP) is still computationally intractable. A Hierarchical gait—centroidal—wholebody pipeline is commonly employed to reduce the planning complexity. However, additional constraints projecting wholebody information to the centroidal level are required to ensure feasible solutions. In practice, these constraints are often computationally expensive and difficult to define, especially when involving angular momentum [1]. On the contrary, [1], [2] adopt a distributed approach, updating centroidal and whole-body models in an alternating fashion. However, contact sequences still need to be pre-specified, and results are only shown for simple walking motions. We improve upon the existing distributed framework by introducing a centroidal optimization capable of discovering both contact sequences and angular momentum trajectories. A new set of consensus constraints is also formulated. Lastly, we evaluate different acceleration techniques to improve the global convergence of the proposed distributed optimization.

### II. METHODS

# A. Centroidal and Whole-Body Optimization

The proposed centroidal optimization utilizes phase-based gait parameterization [3] and equimomental-ellipsoid-based moment of inertia (MoI) parameterization [4]. By explicitly treating phase timings, ellipsoid axes and orientation as decision variables, we can simultaneously solve for footholds, contact sequence/timings, centroidal and momentum trajectories without any knowledge of joint configurations. Using composite rigid body MoI (CRBMoI) as feedback, the ellipsoid MoI tracks the effects of the joint motion on CRBMoI, thus allowing the optimization to produce high-fidelity angular momentum along the time horizon. The dimensionality of the problem is relatively small and it exhibits moderate complexity when transcribed into a NLP through direct collocation.

Leveraging full dynamics such as the angular momentum of limbs is critical for generating dynamic motions. To efficiently solve the whole-body optimization, the locomotion problem is formulated in a hybrid fashion to track the consensus quantities from centroidal optimization, and then solved via Differential Dynamic Programming (DDP).

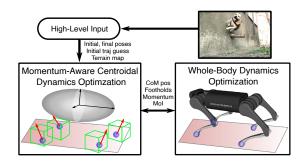


Fig. 1. The consensus framework given a high-level user input.

## B. Distributed Optimization and Convergence Consideration

Different from [1], [2], the consensus between our momentum-aware centroidal and whole-body optimization is enforced by adding equality consistency constraints for Center of Mass (CoM) positions, momentum, footholds, and MoI generated from each optimization. We explore the Alternating Direction Method Multiplier (ADMM) framework [2] to alternately update these two optimization problems as shown in Fig. 1. However, a major defect of standard ADMM algorithms is their slow convergence rate that leads to unsatisfactory accuracy. An accelerated version of ADMM is proposed to speed up the global convergence rate.

### III. PROPOSED EXPERIMENTS

A preliminary flat ground jumping example for our quadruped A1 has been tested in simulation to verify the convergence of the whole framework. Further experiments will focus on designing complex dynamic motions such as parkour or front-flipping to demonstrate the advantages of variable inertia and dynamics consensus. We hypothesize that our framework will show higher agility and trajectory fidelity compared with a hierarchical pipeline.

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