Hybrid Invariant Extended Kalman Filtering for Legged Locomotion on Dynamic Rigid Surfaces

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I. Background

Real-world applications of legged locomotion, such as delivery and courier services, home assistance, and emergency response, demand accurate, reliable state estimation. Although previous researchers have extensively studied state estimators for legged robots on stationary rigid surfaces, few works have investigated the estimator design for dynamic rigid surfaces (DRSs) (i.e., rigid surfaces that move in the inertial frame) such as elevators, ships, public transport vehicles, and airplanes. State estimation of legged locomotion on a DRS remains a challenging problem due to the nonlinear, hybrid nature of walking robot dynamics and nonstationary contact points.

II. Method

In this study, we extend the current invariant extended Kalman filter (InEKF) framework for legged robots [1] to create the hybrid InEKF (HInEKF) for locomotion on a DRS. We choose to build the proposed filter design upon the InEKF framework because of its attractive properties, e.g., the error dynamics are independent from the estimated state, which lead to rapid and guaranteed error convergence.

There are three major contributions of this work. First, We have provably extended the InEKF method to a general class of nonlinear hybrid systems with state-triggered jumps by explicitly formulating the covariance matrix propagation at jumps using saltation matrices [2] on Lie groups.

Second, we have characterized a class of hybrid dynamical systems whose deterministic error jump maps on Lie groups are identity under the assumption of instantaneous jump detection. An appealing feature of such systems is that the invariant error becomes continuous at any jump. Also, the Jacobian matrix of the jump map becomes identity, which simplifies the computation of the saltation matrix for covariance propagation at a jump. This class of hybrid dynamical systems satisfies the following condition:

$$\begin{aligned} \mathbf{\Delta}(\mathbf{ab}) &= \mathbf{a}\mathbf{\Delta}(\mathbf{b}) \ \text{(right-invariant)}; \\ \mathbf{\Delta}(\mathbf{ab}) &= \mathbf{\Delta}(\mathbf{a})\mathbf{b} \ \text{(left-invariant)}, \end{aligned} \tag{1}$$

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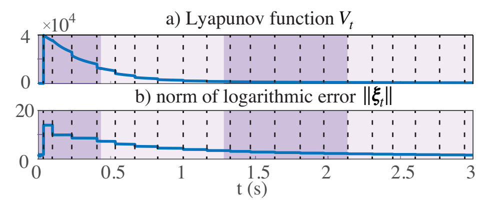

Fig. 1. One trial of bipedal robot walking on the vertically moving surface under HInEKF: a) Lyapunov function and b) norm of logarithmic error $\|\boldsymbol{\xi}_t\|$. The jump times are indicated by the edges of different background colors. Black dash line indicate the timing of LiDAR measurement.

where $\mathbf{\Delta} : G \to G$ is the jump map with G the matrix Lie group, and $\mathbf{a}, \mathbf{b} \in G$.

Third, we have implemented the proposed HInEKF through simulations of bipedal walking on a DRS. The implementation does not require sensing of the robot's base velocity, and it does not assume zero velocities of contact points. Instead, the contact-point velocity is treated as the input to the process model, which is obtained based on: a) the relative position of the robot's support feet with respect to the surface, which is returned by a camera mounted on the robot and b) the surface movement, which is assumed to be provided by the motion monitoring system of the surface (e.g., the navigation system of ships and airplanes).

III. Results and Discussions

We have implemented the proposed HInEKF on a planar bipedal robot walking on a vertically moving rigid surface with noisy sensors including IMU, joint encoders, and LiDAR. Our result (Fig. 1) shows accurate estimation and rapid error convergence without sudden error expansion at jumps. The result demonstrates that the proposed filter could provide a controller [3] with reliable knowledge of the robot's state.

References

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